Challenge in QCD

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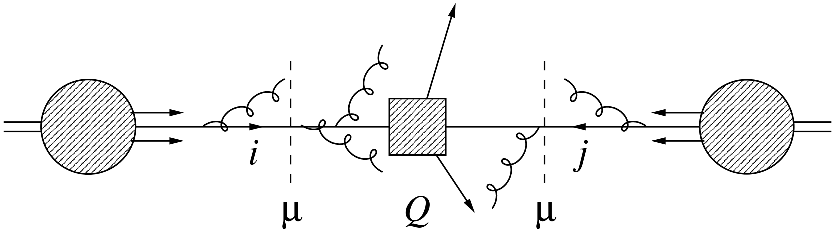
The challenge

- The challenge is to provide the most accurate information possible to experimenters working at the Tevatron and the LHC.
- Proton (anti)proton collisions give rise to a rich event structure.
- Complexity of the events will increase as we pass from the Tevatron to the LHC.
- The goals
  - ⋆ To provide physics software tools which are both flexible and give the most accurate representations of the underlying theories.
  - ⋆ To discover new efficient ways of calculating in QCD.

Hadron-hadron processes

- In hard hadron-hadron scattering, constituent partons from each incoming hadron interact at short distance (large momentum transfer Q^2).



- For hadron momenta P_1, P_2 ($S = 2P_1 \cdot P_2$), form of cross section is

$$\sigma(S) = \sum_{i,j} \int dx_1 dx_2 D_i(x_1, \mu^2) D_j(x_2, \mu^2)$$
$$\hat{\sigma}_{ij}(\hat{s} = x_1 x_2 S, \alpha_S(\mu^2), Q^2/\mu^2)$$

where μ^2 is factorization scale and $\hat{\sigma}_{ij}$ is subprocess cross section for parton types i, j.

Hadron-hadron processes II

- Short distance cross section $\hat{\sigma}_{ij}$ is calculable as a perturbation series in α_S.
- Notice that factorization scale is in principle arbitrary: affects only what we call part of subprocess or part of initial-state evolution (parton shower).
- Unlike e^+e^- or ep, we may have interaction between spectator partons, leading to *soft underlying event* and/or *multiple hard scattering*.

Short-distance cross section

- Tree graph level
  - ⋆ Madgraph/Helas
  - ⋆ Alpgen
  - ⋆ Analytic calculation including CSW tricks
- NLO
  - ⋆ MCFM and NLOJET++
- NLO + parton shower
  - ⋆ MC@NLO
- NNLO
  - ⋆ Drell-Yan Luminosity monitor

The role of tree graphs

- Problems with tree graphs
  a) Overall normalization is uncertain,
  For example, W+4 jets is O(α_S^4), If scale uncertainty changes α_S by 10%, this leads to 40% uncertainty in cross section.
  b) If we wish talk about hadrons, we must apply fragmentation.
  To use universal fragmentation, we must evolve to a fixed scale.
  Tree graphs require a procedure to combine with parton showers.
  c) Sometimes a new parton process appears at NLO, leading to large change in shapes.
- W, Z + n jets known at tree graph level.
  Madgraph II can generate processes with $\leq$ 9 external particles (madgraph.hep.uiuc.edu)
  Vecbos, W-boson plus up to 4 jets or a Z-boson plus up to 3 jets (theory.fnal.gov/people/giele/vecbos.html)
  Alpgen, W,Z + up to 6 jets

Madgraph/Madevent

Stelzer and Maltoni,hep-ph/0208156

- Madgraph II can generate processes with ≤ 9 external particles
- Madevent uses single diagram enhanced multi-channel integration

$$f = \sum_{i=1}^{n} f_i \qquad \text{with} \qquad f_i \geq 0\,, \qquad \forall\ i \quad f_i = \frac{|A_i|^2}{\sum_i |A_i|^2} |A_{\rm tot}|^2\,,$$

where A_i is the amplitude corresponding to a single Feynman diagram. The peak structure of each f_i can be efficiently mapped by a single channel g_i.

- The integration of f reduces to:

$$I \;\; = \;\; \int d\vec{\Phi} f(\vec{\Phi}) = \sum_{i=1}^{n} \int d\vec{\Phi}\, g_i(\vec{\Phi})\, \frac{f_i(\vec{\Phi})}{g_i(\vec{\Phi})} = \sum_{i=1}^{n} I_i\,,$$

Spinor techniques (analytic results)

- Denote spinor for lightlike vectors as follows:-

$$
\begin{aligned}
|k+\rangle &= \text{right-handed spinor for massless vector } k \\
|k-\rangle &= \text{left-handed spinor for massless vector } k
\end{aligned}
$$

- Polarization vectors are given by ($q \equiv$ gauge choice)

$$
\varepsilon_\mu^+ = \frac{\langle q^- |\gamma_\mu| k^- \rangle}{\sqrt{2}\langle qk\rangle}, \quad \varepsilon_\mu^- = \frac{\langle q^+ |\gamma_\mu| k^+ \rangle}{\sqrt{2}[kq]}
$$

- Obeys all the requirements of a polarization vector

$$
\varepsilon_i^2 = 0, \quad k \cdot \varepsilon(k,q) = 0, \quad \varepsilon^+ \cdot \varepsilon^- = -1
$$

- Equivalent notations

$$
\epsilon^{ab}\lambda_{ja}\lambda_{lb} \equiv \langle jk\rangle \equiv \langle k_j^- | k_l^+ \rangle = \sqrt{2k_j \cdot k_l}\, e^{i\phi}
$$

$$
\epsilon^{\dot{a}\dot{b}}\tilde{\lambda}_{j\dot{a}}\tilde{\lambda}_{l\dot{b}} \equiv [jk] \equiv \langle k_j^+ | k_l^- \rangle = -\sqrt{2k_j \cdot k_l}\, e^{-i\phi}
$$

MHV amplitudes

- Consider the 5 gluon amplitude
- Decompose gluonic amplitude into color-ordered sub-amplitudes

$$A = \mathrm{Tr}\{t^{a1}t^{a2}t^{a3}t^{a4}t^{a5}\}m(1,2,3,4,5) + \text{permutations}$$

- Two of the color stripped amplitudes vanish

$$\begin{aligned} m(g_1^+, g_2^+, g_3^+, g_4^+, g_5^+) &= 0 \\ m(g_1^-, g_2^+, g_3^+, g_4^+, g_5^+) &= 0 \end{aligned}$$

- The maximal helicity violating 5 gluon amplitude

$$m(g_1^-, g_2^-, g_3^+, g_4^+, g_5^+) = \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 51\rangle}$$

$\langle ij\rangle, [ij]$ useful because QCD amplitudes have square root singularities

MHV amplitudes

Parke and Taylor, Berends and Giele

- The generalization to the case with two contiguous positive helicity gluons and $n-2$ negative gluons is

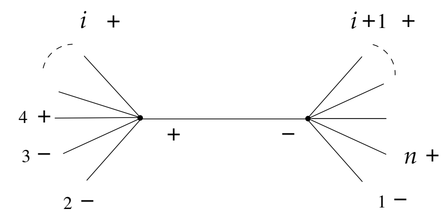
$$m(g_1^-, g_2^-, g_3^+, \ldots g_n^+) = \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \ldots \langle n1 \rangle}$$

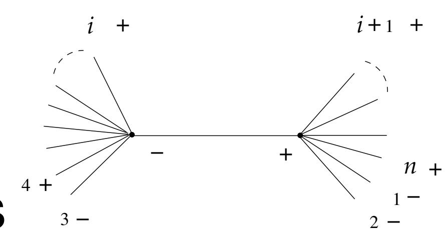
- Remember $\langle ij \rangle$ are the spinor products $\sim \sqrt{(2p_i \cdot p_j)}$

CSW

Cachazo,Svrcek,Witten

- Motivated by a topological string theory in twistor space, use MHV amplitudes as effective vertices to build more complicated amplitudes
- Use MHV amplitudes as effective vertices to build more





  complicated amplitudes
- Obtain simple expressions for tree amplitudes in terms of spinor products
- Extension to loops?

MHV2

- Define an offshell MHV vertex using the QCD Parke-Taylor amplitude.

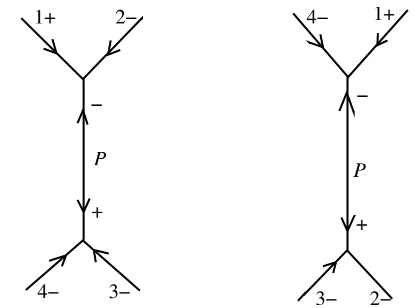
$$V(1^-,2^-,3^+,\ldots,n^+,P^+)=\frac{\langle 12\rangle^4}{\langle 12\rangle\ldots\langle n-1,n\rangle\langle n,P\rangle\langle P1\rangle}$$

- Continue the spinor off-shell $\langle iP\rangle=\eta\sum_{j=1}^{n}\langle i^-|\not{k}_j|q^-\rangle$ where $P=k_1+k_2+\ldots k_n$, with lightlike auxiliary q
- Final result independent of η and q
- Easy to sew MHV vertices together to obtain more complicated amplitudes
- n gluon $---+++\ldots++$ amplitude is the sum of MHV diagrams

MHV example, (n=4)

- Consider the two MHV vertex diagrams which give $+---$ gluon amplitude (it vanishes in Yang-Mills theory)

  First diagram



$$m_1(1,2,3,4) = \frac{\langle 2P\rangle^4}{\langle 12\rangle\langle 2P\rangle\langle P1\rangle}\frac{1}{P^2}\frac{\langle 34\rangle^4}{\langle 34\rangle\langle 4P\rangle\langle P3\rangle}$$

- According to our continuation this is

$$\frac{\langle 2|(\not{1}+\not{2}|q\rangle^3}{\langle 12\rangle\langle 1|(\not{1}+\not{2}|q\rangle}\frac{1}{\langle 12\rangle[21]}\frac{\langle 34\rangle^3}{\langle 4|\not{3}+\not{4}\rangle|q\rangle\langle 3|\not{3}+\not{4}\rangle|q\rangle} = \frac{[1q]}{[2q][3q][4q]}\frac{\langle 34\rangle}{[21]}$$

- Adding the second diagram ($2\leftrightarrow 4$),

$$m_1(1,2,3,4)+m_1(1,4,3,2) = \frac{[1q]}{[2q][3q][4q][21][41]}(\langle 34\rangle[41]+\langle 32\rangle[21]) = 0$$

MHV outlook

- Lead to beautiful results for gauge theory amplitudes; however the evaluation of pure gluon tree graphs is a numerically solved problem, (Berends-Giele recursion).
- So far impact on real phenomenology limited; simple tree graph results for H-gluon amplitudes Dixon et al, Badger et al
- Extension to loops is the next frontier; the new techniques solve the problem of computing one-loop amplitudes of gluons in $\mathcal{N} = 4$ super Yang-Mills. Will this lead to a comparable simplification of standard model one loop amplitudes?

Why NLO?

The benefits of higher order calculations are:-

- Less sensitivity to unphysical input scales (eg. renormalization scale)
- First prediction of normalization of observables at NLO
- More accurate estimates of backgrounds for new physics searches.
- Confidence that cross-sections are under control for precision measurements
- More physics
  - ⋆ Jet merging
  - ⋆ Initial state radiation
  - ⋆ More species of incoming partons enter at NLO
  - ⋆ It represents the first step for other techniques matching with resummed calculations, eg. NLO parton showers

An experimenter's wishlist

Run II Monte Carlo Workshop

Single Boson	Diboson	Triboson	Heavy Flavour
$W + \leq 5j$	$WW + \leq 5j$	$WWW + \leq 3j$	$t\bar{t} + \leq 3j$
$W + b\bar{b} \leq 3j$	$W + b\bar{b} + \leq 3j$	$WWW + b\bar{b} + \leq 3j$	$t\bar{t} + \gamma + \leq 2j$
$W + c\bar{c} \leq 3j$	$W + c\bar{c} + \leq 3j$	$WWW + \gamma\gamma + \leq 3j$	$t\bar{t} + W + \leq 2j$
$Z + \leq 5j$	$ZZ + \leq 5j$	$Z\gamma\gamma + \leq 3j$	$t\bar{t} + Z + \leq 2j$
$Z + b\bar{b} + \leq 3j$	$Z + b\bar{b} + \leq 3j$	$ZZZ + \leq 3j$	$t\bar{t} + H + \leq 2j$
$Z + c\bar{c} + \leq 3j$	$ZZ + c\bar{c} + \leq 3j$	$WZZ + \leq 3j$	$t\bar{b} \leq 2j$
$\gamma + \leq 5j$	$\gamma\gamma + \leq 5j$	$ZZZ + \leq 3j$	$b\bar{b} + \leq 3j$
$\gamma + b\bar{b} \leq 3j$	$\gamma\gamma + b\bar{b} \leq 3j$		
$\gamma + c\bar{c} \leq 3j$	$\gamma\gamma + c\bar{c} \leq 3j$		
	$WZ + \leq 5j$		
	$WZ + b\bar{b} \leq 3j$		
	$WZ + c\bar{c} \leq 3j$		
	$W\gamma + \leq 3j$		
	$Z\gamma + \leq 3j$		

NLO calculation

- Ingredients in a NLO calculation are
  - ⋆ Born level amplitude
  - ⋆ Real contribution: Addition of one extra parton to Born level process
  - ⋆ Virtual contribution: Interference of one-loop amplitude with Born amplitude
- Real and virtual separately contain singularities from the soft and collinear regions which cancel in the sum.
- Calculation of one loop amplitude rapidly becomes complicated as number of partons increases.
- Especially true as we go beyond the most symmetric cases with all gluons.

MCFM overview

John Campbell and R.K. Ellis

- Parton level cross-sections predicted to NLO in α_S

$p\bar{p} \to W^{\pm}/Z$	$p\bar{p} \to W^{+} + W^{-}$
$p\bar{p} \to W^{\pm} + Z$	$p\bar{p} \to Z + Z$
$p\bar{p} \to W^{\pm} + \gamma$	$p\bar{p} \to W^{\pm}/Z + H$
$p\bar{p} \to W^{\pm} + g^{\star}\ (\to b\bar{b})$	$p\bar{p} \to Zb\bar{b}$
$p\bar{p} \to W^{\pm}/Z + 1$ jet	$p\bar{p} \to W^{\pm}/Z + 2$ jets
$p\bar{p}(gg) \to H$	$p\bar{p}(gg) \to H + 1$ jet
$p\bar{p}(VV) \to H + 2$ jets	$p\bar{p} \to t + X$

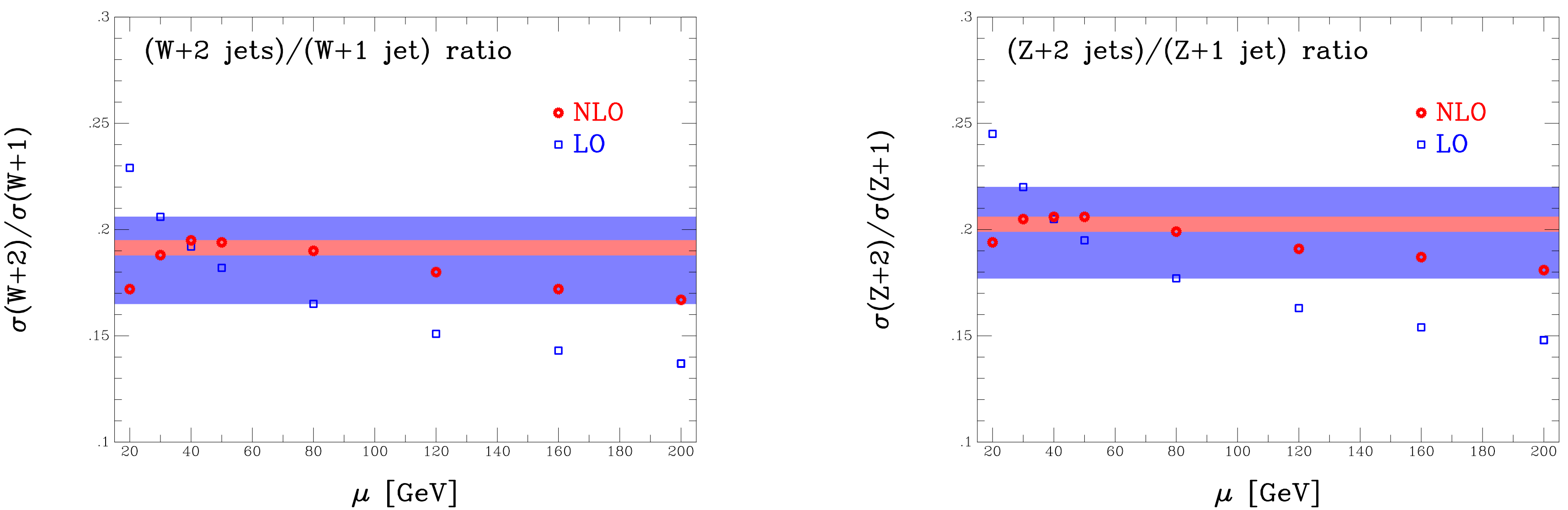
- $\ominus$ low particle multiplicity (no showering)
- $\ominus$ no hadronization
- $\ominus$ hard to model detector effects
- $\oplus$ less sensitivity to μ_R, μ_F
- $\oplus$ rates are better normalized
- $\oplus$ fully differential distributions

MCFM Information

- Version 4.1 (January 05) available at:
  http://mcfm.fnal.gov
- Improvements over previous releases:
  - ⋆ more processes ($Z + b$, single top, $\ldots$)
  - ⋆ better user interface
  - ⋆ support for PDFLIB, Les Houches PDF accord
    ($\longrightarrow$ PDF uncertainties)
  - ⋆ ntuples as well as histograms
  - ⋆ unweighted events
  - ⋆ Pythia/Les Houches generator interface (LO)
  - ⋆ separate variation of factorization and renormalization scales
  - ⋆ 'Behind-the-scenes' efficiency

$W/Z+$ jet cross-sections

- The $W/Z+2$ jet cross-section has only recently been calculated at NLO and should provide an interesting test of QCD (cf. many Run I studies using the $W/Z+1$ jet calculation in DYRAD)
- For instance, the theoretical prediction for the number of events containing 2 jets divided by the number containing only 1 is greatly improved.



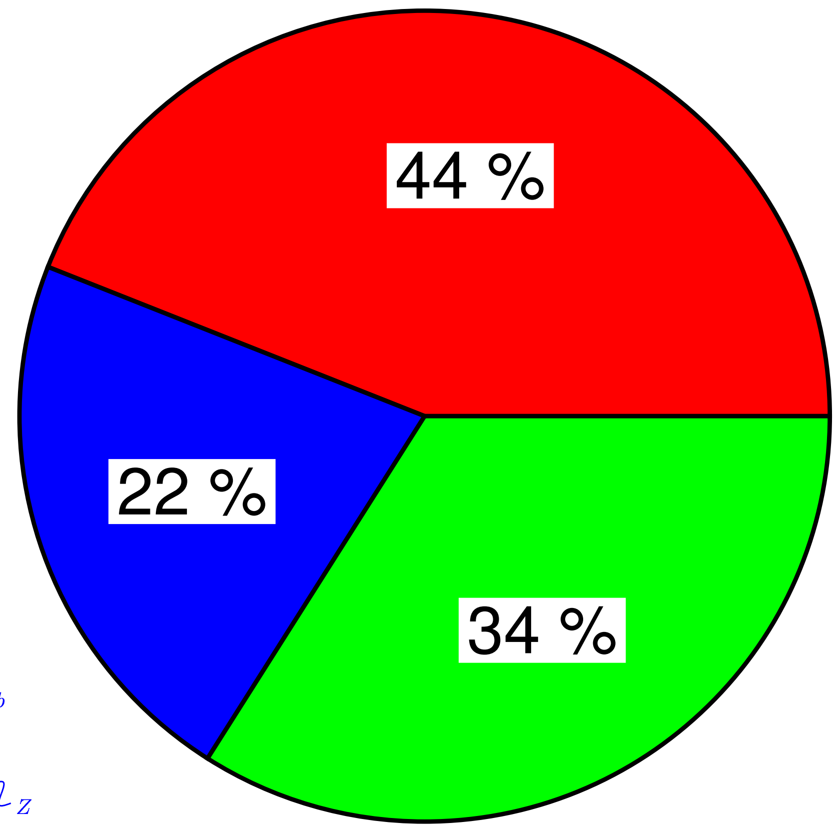
$Z + b$ at NLO - Run II

Campbell et al, hep-ph/0312024

- $p_T^{\rm jet} > 15$ GeV, $|\eta^{\rm jet}| < 2$
- $\sigma(Z + \text{ one } b \text{ tag}) = 20$ pb
- Fakes from $Z+$ jet events are significant
- Prediction for ratio of $Z + b$ to untagged $Z + \text{jet}$ is

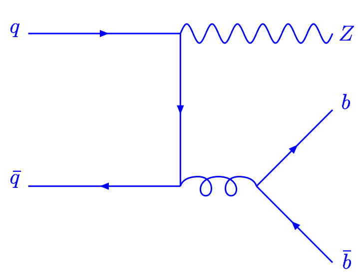
  0.02 ± 0.004

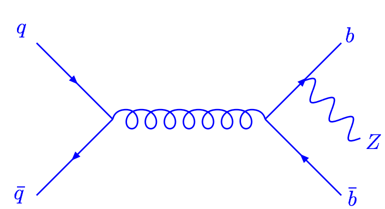
$gb \to Zb$



$q\bar{q} \to Z(b\bar{b})$

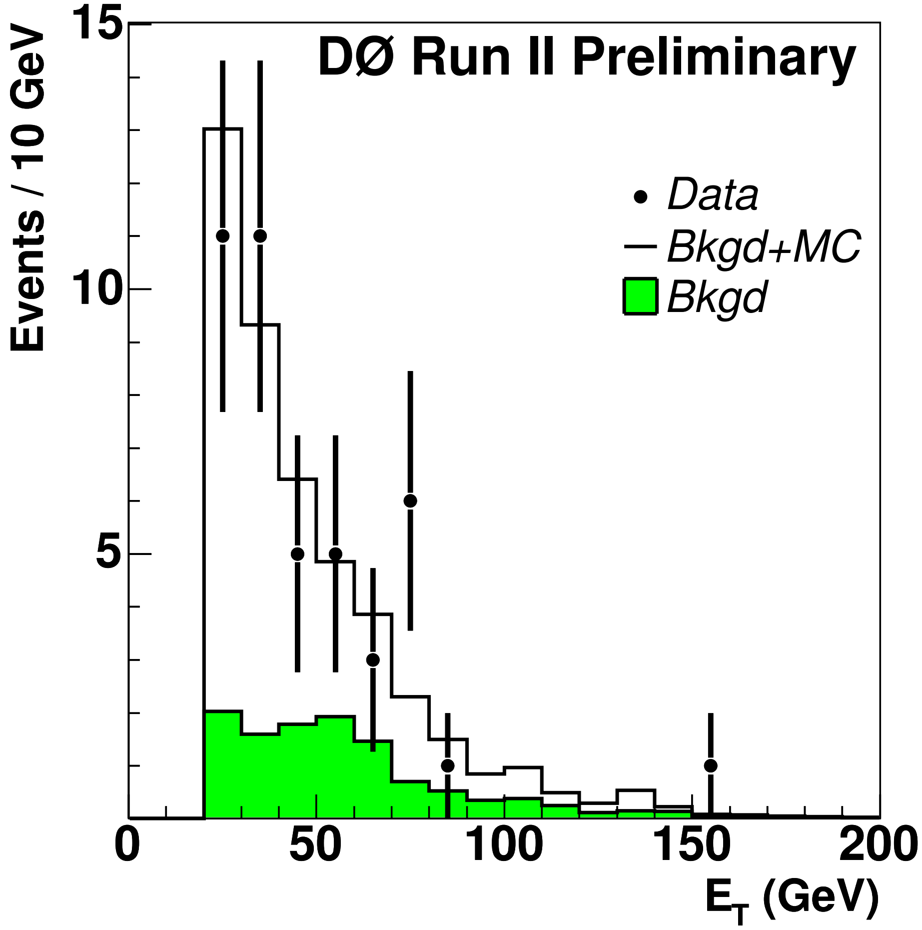
$Z + 1$ jet (fake rate of 1%)





Experimental result

- Based on $189 \ \mathrm{pb}^{-1}$ of data from Run II



Preliminary ratio of cross-sections:

$$\frac{\sigma(Z+b)}{\sigma(Z+j)} = 0.024 \pm 0.07$$

compatible with the NLO prediction

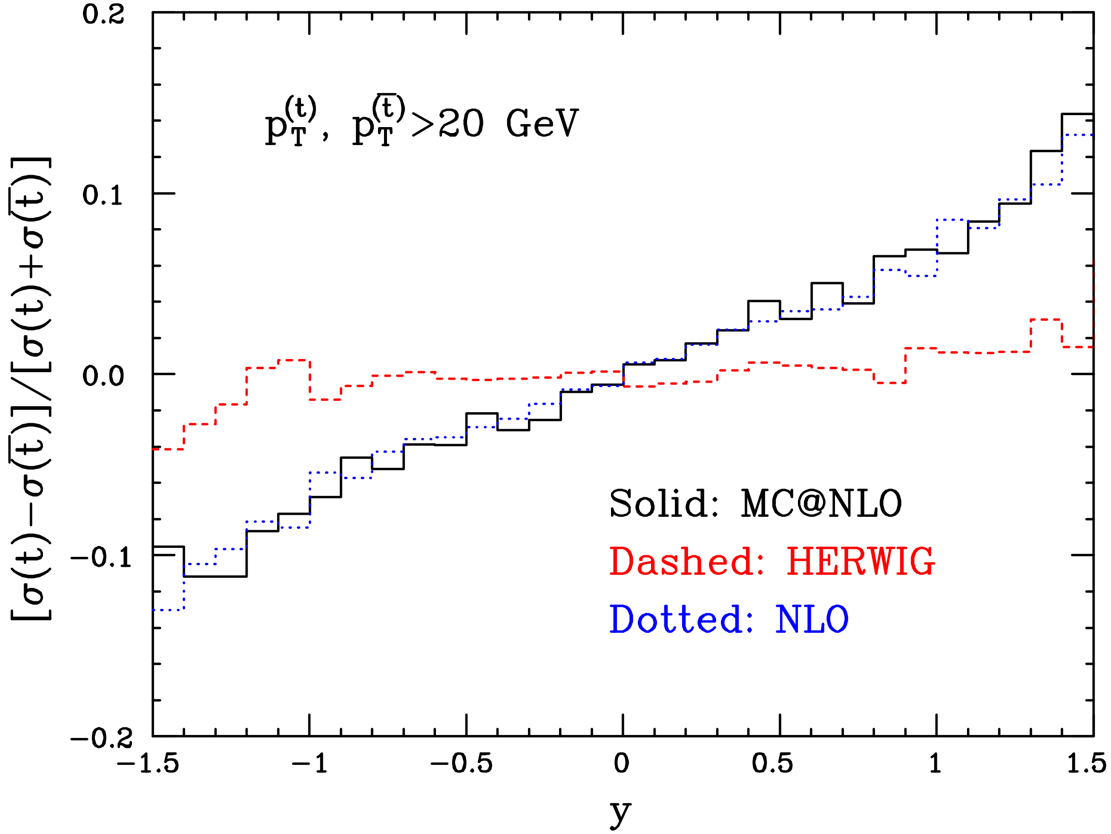
Can one improve on NLO?

Frixione et al,hep-ph/0305252,hep-ph/0204244

- www.hep.phy.cam.ac.uk/theory/webber/MCatNLO/
- Relies on the appropriate NLO process having been calculated.
- Output is a set of events, which are fully inclusive
- Total rates are accurate to NLO
- NLO results for all observables are recovered upon expansion in α_S
- Currently a limited number of available processes, Higgs boson,
  single vector boson, W/Z,
  vector boson pair, WW,
  heavy quark pair, $Q\bar{Q}$
  lepton pair production, e^+e^-

Asymmetry in top production

Frixione,Nason,Webber



- MC@NLO

Why NNLO

- reduced scale dependence
- Event has more partons in the final state and hence closer to the real world
- Better description of transverse momentum of final state due to double radiation off initial states.
- NNLO is the first serious estimate of error.
- obvious application: Reduction of uncertainty in α_s at e^+e^- colliders. Currently: $\alpha_S = 0.121 \pm 0.001(\text{exp}) \pm 0.006(\text{theory})$ (resummed NLO). NNLO would reduce the uncertainty.
- Potent theoretical tool for investigating perturbation theory

The frontier – NNLO

- Number of processes known at NNLO is rather small.
- Processes considered tend to be the most inclusive.
- For more exclusive processes there may be other theoretical uncertainties of the same order as the NNLO contributuions.

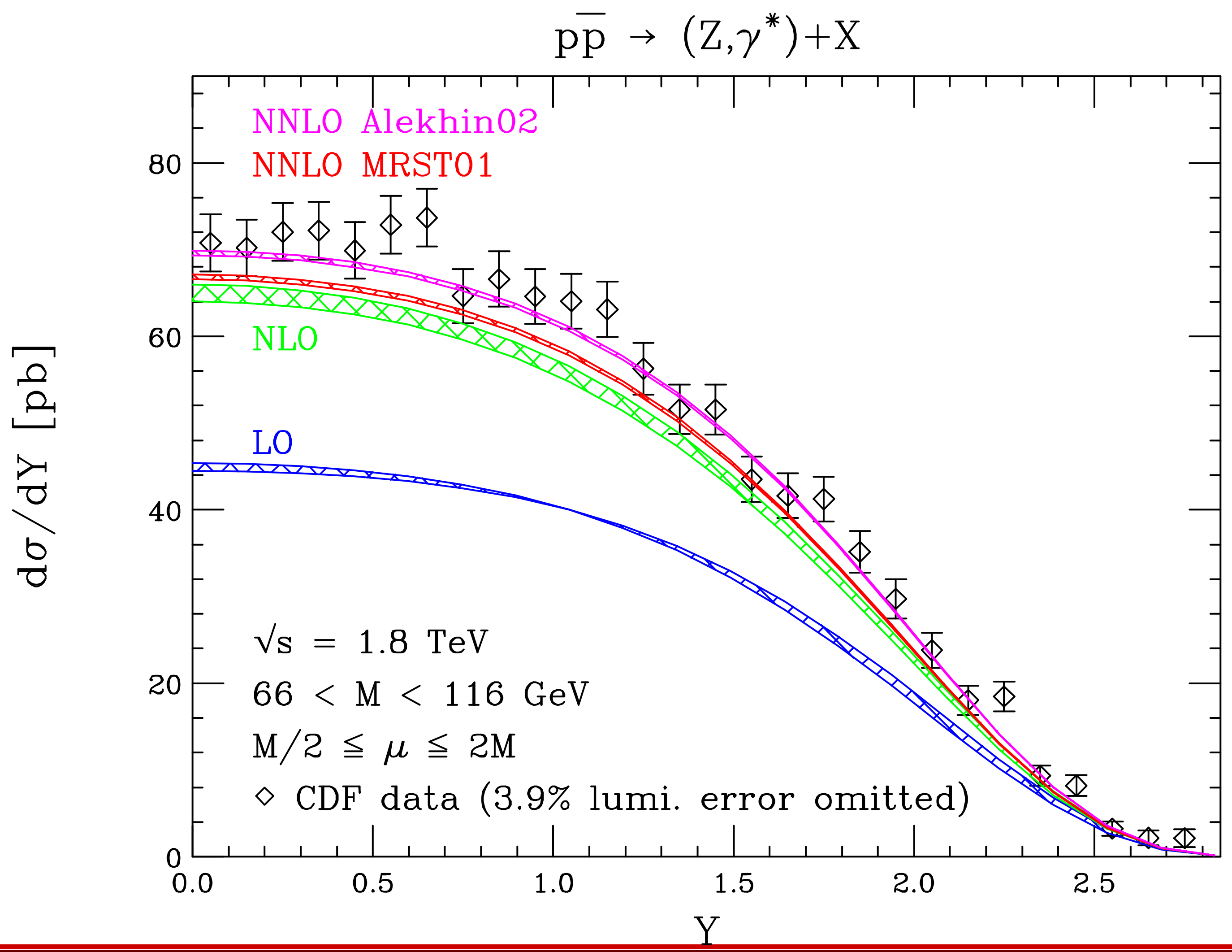
Processes known at NNLO

Stirling

ep	DIS polarised and unpolarised structure function coefficient functions Sum Rules (GLS, Bj, ...) DGLAP splitting functions
e^+e^-	total hadronic cross section, and $Z \to$ hadrons, $\tau \to \nu +$ hadrons heavy quark pair production near threshold C_F^3 part of $\sigma(3 \; \mathrm{jet})$
pp	inclusive W, Z, γ^* inclusive γ^* with longitudinally polarised beams W, Z, γ^* differential rapidity distribution H, A total and differential rapidity distribution WH, ZH
HQ	$Q\overline{Q}$–onium and $Q\bar{q}$ meson decay rates

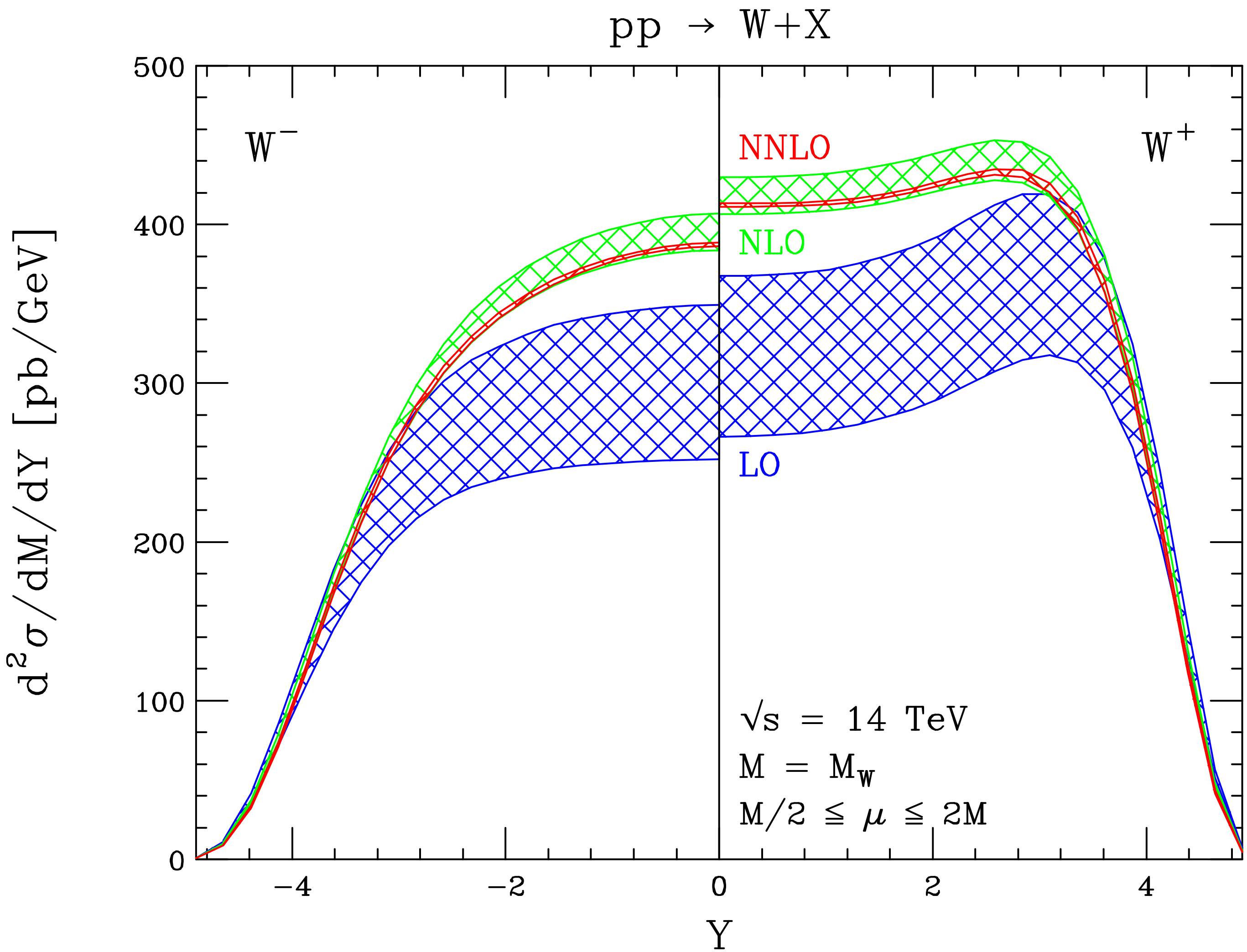
Drell-Yan processes at NNLO

Anastasiou et al.



Luminosity monitor for LHC

Anastasiou et al.



Current research directions

- $W + 3{,}4$ jet cross-sections at NLO
  - ⋆ New technology needed: ready for Run II?

Nagy and Soper, hep-ph/0308127
Giele and Glover, hep-ph/0402152

- Inclusion of b mass effects in $Wb\bar{b}$ and $Zb\bar{b}$
  - ⋆ Technology available: some efforts are underway . . . c.f. $Hb\bar{b}$

W. Beenakker et al., hep-ph/0211352
S. Dawson et al., hep-ph/0311216

- Merging of existing NLO calculations with a parton shower
  - ⋆ Possible: MC@NLO has yet to be applied to $W/Z+$ jets
- Further study of recent ideas regarding improving parton showers (most promising in the short term)
  - ⋆ Matrix elements corrections - CKKW, Krauss et al . . .
- Comparisons of all approaches amongst themselves and with data is important.